Cognitive Science
Knowledge Representation & Organisation

Erasmus Programme
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Predicate Logic

Universe of discourse and typical language
A predicate is a relation with one or more parameters corresponding to a fact:
blue(sky) : the sky is blue
sky is a constant
csg (C,S,G) : Course-Student-Grade. Student with the identity S has taken in the course C grade G. For certain values of C,S,G (replacing variables with constants) it is true or false
snap (S, N, A, P) : Student–Name–Address-Phone. Student with identity S has name N, address A and phone number P. For certain values of S, N, A, P (replacing variables with constants) it is true or false
human(X) : X is human
mortal (X) : X is mortal
C, S, G, S, N, A, P, X are variables which can be substituded by constants

Quantifiers

Universal quantifier

$\forall X \, (\, man(X) \rightarrow mortal(X) \,)$

Existential quantifier

$\exists X \, (\, year(X) \rightarrow leap(X) \,)$

Deduction Rules

1. Application of Modus Ponens
human(a) → mammal(a)
human(a)
We conclude:
mammal(a)
2. Incorrect application of Modus Ponens
human(a) → mammal(a)
mammal(a)
We conclude:
human(a) [NOT VALID!!!]
3. Application of Modus Tollens
human(a) → mammal(a)
NOT mammal(a)
We conclude (M.T.):
NOT human(a)

Substitution & M.P.

Given propositions:
If something (X) is human then this is mortal
$(\forall X)$ (human(X) $\rightarrow$ mortal(X)) (prop.1)
Vincent is human
human(vincent) (prop.2)
To be proved:
Vincent is mortal
mortal (vincent) (prop.3)
(prop.1) substitute X with vincent:
human(vincent) $\rightarrow$ mortal(vincent) (prop.4)
From (prop.4) and (prop.2) through M.P. we conclude that:
mortal(vincent)

Substitution & M.P. (cont)

Given:
Every human is animal
Every greek is human
Markos is greek
To be proved:
Markos is animal

Written in Predicate Logic
$(\forall X)\ (h(X) \rightarrow a(X))$ (prop.1)
$(\forall X)\ (g(X) \rightarrow h(X))$ (prop.2)
g(markos) (prop.3)
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a(markos)

Proof
(prop.2) substitute X with markos
$g(markos) \rightarrow h(markos)$ (prop.4)
From (prop.4) and (prop.3) through M.P. we conclude that:
h(markos) (prop.5)
(prop.1) substitute X with markos
$h(markos) \rightarrow a(markos)$ (prop.6)
From (prop.6) and (prop.5) through M.P. we conclude that:
a(markos)

Exercise-3a

Given propositions:
If something (X) is cat then this is feline
Eleanor is cat
To be proved:
Eleanor is feline

The proof must be presented by using the deduction rule: Modus Ponens (M.P.) along with substitution

Exercise-3b

- Someone who gets high degree in medicine becomes good physician
- A good physician cures the patients
- Anne got high degree in medicine
- To be proved
- Anne cures the patients

De Morgan rules

- $\neg (p \vee q) \mathrel{|}\!\!=\!\!\mathrel{|} \neg p \wedge \neg q$
- $\neg (p \wedge q) \mathrel{|}\!\!=\!\!\mathrel{|} \neg p \vee \neg q$
- $\neg(\forall X)\ (p(X)) \mathrel{|}\!\!=\!\!\mathrel{|} (\exists X)\ (\neg p(X))$
- $\neg(\exists X)\ (p(X)) \mathrel{|}\!\!=\!\!\mathrel{|} (\forall X)\ (\neg p(X))$

Resolution principle: Elimination of quantifiers

- We can eliminate universal quantifier by substituting the variable name with a new name, uniquely defined for each proposition.
- We can eliminate existential quantifier by substituting the variable name with an assumed constant corresponding to a certain object of the universe of discourse, uniquely defined for each proposition.

Proof in Predicate Calculus through Resolution Principle

- Aristotelian scheme
- Given propositions:
  - Every human is animal
  - Every Greek is human
- To be proved:
  - Every Greek is animal
- Step 1
  - $(\forall X)\ (h(X) \rightarrow an(X))$ (p1)
  - $(\forall X)\ (g(X) \rightarrow h(X))$ (p2)
  - $\neg\ ((\forall X)\ (g(X) \rightarrow an(X))\ )$ (p3)

Proof in Predicate Calculus through Resolution Principle (cont)

- Step 2
  - $(\forall X)\ (\neg h(X) \lor an(X))$ (p1)
  - $(\forall X)\ (\neg g(X) \lor h(X))$ (p2)
  - $(\exists X)\ \neg\ (\ g(X) \rightarrow an(X)\ )$ |=|
  - $(\exists X)\ \neg\ (\neg g(X) \lor an(X)\ )$ |=|
  - $(\exists X)\ (\ g(X) \land \neg an(X)\ )$

Proof in Predicate Calculus through Resolution Principle (cont)

- Step 3
  - $\neg h(X1) \vee an(X1)$ (p1)
  - $\neg g(X2) \vee h(X2)$ (p2)
  - $g(a) \wedge \neg an(a)$ |=|
  - $g(a)$ (p3)
  - $\neg an(a)$ (p4)
- Step 4
  - (p1) and (p4) for $X1=a \Rightarrow \neg h(a)$ (p5)
  - (p2) and (p3) for $X2=a \Rightarrow h(a)$ (p6)
  - (p5) and (p6) $\Rightarrow$ 0 (αντίφαση)

Proof through Resolution Principle –Exercise 4

- If every M are P and some S are M ⇨ some S are P".
- Each m.p. is of high salary
- $(\forall X)\ (mp(X) \rightarrow hs(X))$ (prop1)
- Some lawyers are m.p
- $(\exists X)\ (l(X) \wedge mp(X))$ (prop2)
- Conclusion (to be proved)
- Some lawyers are of high salary
- $(\exists X)\ (l(X) \wedge hs(X))$ (prop3)
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